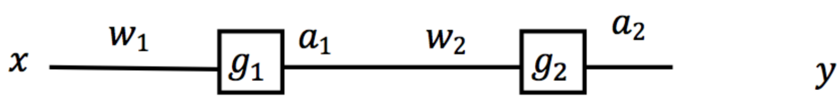
CS188 Fall 2016: Discussion 11

Note: The original worksheet was buggy. Here's the fixed version.

Neural Nets and Computation Graphs

Consider the following two-neuron network for binary classification:



Here x is a single real-valued input (not a vector) with an associated class y (0 or 1). There are two neurons, with input weights w_1 and w_2, and activation functions g_1 and g_2. The output

$h_w(x) = a_2$

is a value between 0 and 1, representing the probability of being in class 1. We will be using a real-valued loss function $Loss_w(x, y)$.

Q1**:**

Let z_1 and z_2 refer to the pre-activation values at neuron 1 and neuron 2, repsecitvely. Write z_1, a_1, z_2, and a_2 in terms of the previous values of the neural network**.**

Q2:

Write the output a_2 in terms of the input x, weights w_i , and activation functions g_i.

Q3:

Suppose the loss function is quadratic: $(Loss_w(x, y) = (y - a_2)^2)$. Draw the computational graph for the loss function in terms of w_1, w_2, x, y, z_1, a_1, z_2, and a_2.

Q4:

Use the chain rule to derive $\partial Loss/\partial w_2$. Write your expression as a product of partial derivatives that can be directly computed – you don't have to directly compute them. (Hint: the series of expressions you wrote in part 1 will be very useful; you may use any of those variables. Also use the graph from Q3)**.**

Q5:

Now use the chain rule to derive $\partial Loss/\partial w_1$ in terms of the same quantities as Q4.

Q6:

Suppose the loss function is quadratic $(Loss_w(x, y) = (y - a_2)^2)$ and g_1 and g_2 were both sigmoid functions $1/(1 + e^{-z})$. Using the fact that $\partial g_i/\partial z_i = g_i(z_i)(1 - g_i(z_i))$, write $\partial Loss/\partial w_2$ and $\partial Loss/\partial w_1$ in terms of x, y, w_i , a_i , and z_i**.**

Q7:

Write the stochastic gradient descent update for w_1 in terms of the step size α and the values computed above.

Q8:

True or False: For this classifier, there exists some value S for which $x < S$ is classified as belonging to class 0, and $x > S$ is classified as belonging to class 1.