TA: Sherdil Niyaz

CS61B Extra Problems 5 Solutions

I Hope You Like Math…

Using big-O notation.

(a) For each pair of functions $f(n)$ and $g(n)$, state whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. For example, for $f(n) = n^2$ and $g(n) = 2n^2 - n + 3$, write $f(n) = \Theta(g(n))$.

- $f(n) = n$ and $g(n) = n^2 - n$
- $f(n) = n^2$ and $g(n) = n^2 + n$
- $f(n) = 8n$ and $g(n) = n \log n$
- $f(n) = 2^n$ and $g(n) = n^2$
- $f(n) = 3^n$ and $g(n) = 2^{2n}$

(b) For each of the following functions $f(n)$, state the order of growth using Θ notation. For example, for $f(n) = n + 5$ write $f(n) = \Theta(n)$.

- $f(n) = 50$
- $f(n) = n^2 - 2n + 3$
- $f(n) = n + \cdots + 2 + 1$
- $f(n) = n^{100} + 1.01^n$
- $f(n) = n^{1.1} + n \log n$

Answers:

(a)
- $f(n) = O(g(n))$
- $f(n) = \Theta(g(n))$
- $f(n) = O(g(n))$
- $f(n) = \Omega(g(n))$
- $f(n) = O(g(n))$

(b)
- $f(n) = \Theta(1)$
- $f(n) = \Theta(n^2)$
- $f(n) = \frac{(n+1)n}{2} = \Theta(n^2)$
- $f(n) = \Theta(1.01^n)$
- $f(n) = \Theta(n^{1.1})$

Code Analysis

For the pseudo-code snippet below, give the asymptotic running time in Θ notation. Assume that basic arithmetic operations (+,−,×, and /) are constant time.

```
for i := 1 to n do
    j := 0;
    while j ≤ i do
        j := j + 2
```

Answer:

The inner loop takes time $i/2$, so the running time is

$$\sum_{i=1}^{n} i/2 = \Theta(n^2).$$

Credits:

Problems **shamelessly and completely** stolen from CS 170 Discussion 1, Fall 2015.