UC Berkeley – Computer Science
CS61B: Data Structures

Practice Midterm #2, Spring 2016 **Solutions**
Constructed by Sherdil Niyaz
(Please email me with any typos or other mistakes that you find)

This practice test has ? questions worth a total of ?? points. This exam has been written by a TA who has **not** yet looked at the official Midterm 2. This is an unofficial practice test to give you *practice* for the actual midterm. There's a very high chance your midterm will look nothing like this. This is only a tool to give you practice and tell you what you need to study.

*"I pledge to solve all problems on this practice exam **without** looking at the solutions, and to never be discouraged while studying!"*

Signature: _____

Tips:

- There may be partial credit for incomplete answers on the actual exam. Write as much of the solution as you can, but bear in mind that we may deduct points if your answers are much more complicated than necessary.
- There are a lot of problems on this exam. Work through the ones with which you are comfortable first. Do not get overly captivated by interesting design issues or complex corner cases you're not sure about.
- Not all information provided in a problem may be useful.
- Unless otherwise stated, all given code on this exam should compile. All code has been compiled and executed before printing, but in the unlikely event that we do happen to catch any bugs in the exam, we'll announce a fix. Unless we specifically give you the option, the correct answer is not 'does not compile.'
- Ask Professor Hug to give you a hug at the end of the semester.

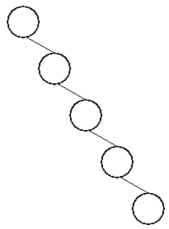
Optional. Mark along the line to show your feelings on the spectrum between ☹ and ☺.
Before exam: [☹_____☺].
After exam: [☹_____☺].

1. **Tree Huggers**

a) If we insert n items into a BST, and these items are in **sorted** order (from least to greatest), what is the **total** runtime of all n of these insertions? **Why?** Draw what the tree looks like now.

Solution:
The total running time for all n insertion operations together is in Θ(n^2). Note that the nth insertion takes n units of work since we have traverse the entire tree each time and then insert the item. We end up with something resembling the sum 1 + 2 + ….. + n, which we saw in lecture goes to Θ(n^2).

By the way, tree looks really "spindly":

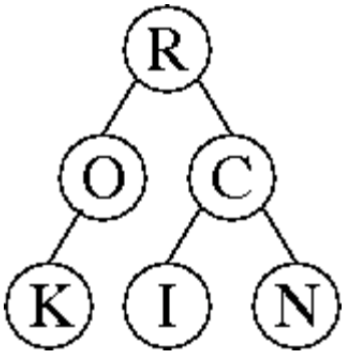


b) If we now removed all **n** of these entries in the BST in **sorted order** (from least to greatest), what is the runtime of all **n** of these operations? **Why?**

Solution:
The total running time for all n removal operations together is in Θ(n), because each remove() operation removes a root that has only one child, which is done in constant time.

c) Fill in the following tree so its **postorder** traversal is K O I N C R.

Solution:



2. **Asymptotic Analyses**

a) Rewrite the following expression in the **simplest possible** form, using O notation.

$$O(3^{2x} + 18log_4(x^2z^4) + max\{y\sqrt{log(z)},\ x^{12}\} + 7y^2/z^2 + 8)$$

Solution:

$$O(9^x + \log z + y\sqrt{\log z} + y^2 / z^2).$$

b)

(5 points) For each of the following statements, write "true" or "false." If you write "false," give a counterexample—functions $f(n)$ and $g(n)$ that show the statement is false, and an explanation why that counterexample shows the statement is false. If you write "true," use the definition of big-Oh to prove that the statement is true. You may assume that $g(n) \to \infty$ as $n \to \infty$. (Note: you won't get credit for the true/false answer; only for the explanation.)

$$\text{if } f(n) \in O(g(n)), \textbf{ then } 2^{f(n)} \in O(2^{g(n)})$$

Solution:
False. A counterexample is f(n) = 2n; g(n) = n. Clearly f(n) ∈ O(g(n)), but 2^f(n) = 4^n ∉ O(2^n) = O(2^g(n)), because 4^n is larger than 2^n by a factor of 2^n (which is not a constant).

$$\text{if } f(n) \in O(g(n)), \textbf{ then } log\, f(n) \in O(log\, g(n))$$

Solution:
True. We assume that f(n) ∈ O(g(n)), which means that there exist constants c and N such that for all n ≥ N, f(n) ≤ c g(n).
It follows that for all n ≥ N, log f(n) ≤ log c + log g(n). Let N' be a number such that for all n ≥ N', g(n) ≥ c. Then for all n ≥ max{N, N'}, log f(n) ≤ 2 log g(n), which proves that log f(n) ∈ O(log g(n)).

3. **Code Runtime**

a) Consider the following pseudo-code:

```
Power(a,n):
1. If n = 0: return 1.
2. Return a × Power(a,n − 1).
```

What is the runtime, in Θ notation?

Solution:
Θ(n)

Now consider the following, different pseudo-code:

```
AltPower(a,n):
1. If n = 0: return 1.
2. If n = 1: return a.
3. If n is even:
4.     Return AltPower(a × a,n/2).
5. else:
6.     Return a × AltPower(a × a,(n − 1)/2).
```

What is the runtime this time, in Θ notation?

Solution:
Θ(logn)

b) Which one of the two would you think completes faster?

Solution:
The second one.

4. **Dat Hash Doe**

After acing 61B, you just landed an internship at MyFace™, a hip new startup in Silicon Valley! Congratulations on being on your way to **six figs respect**. Unfortunately, the other intern on your team is from that Junior College across the bay. After enjoying all of the free food, nap pods, and complementary rickshaw service that MyFace has to offer, the two of you get into an argument about hashing (once you stop goofing off and actually start working on your project together).

a) Your friend from that "other" school is building a hashmap. For insertion, they take the value produced by calling .hashcode() on some random Java object (you can't assume anything about how .hashcode() works). Your friend then uses this value as the index of the bucket that this random item will map to. Is this correct? It so, explain why. If not, why is it broken, and how would you fix it?

Solution:
This is broken. There's **no** guarantee at all that the values produced by calling .hashcode() on some random object are actual indices that exist in the array we're using for our hashmap. For example, .hashcode() could return 200, but we could only have 20 buckets to work with. We would get an outofbounds exception.

To fix this, your friend should have used a **compression function.**

b) Your friend suggests another, simpler hash function. All items will map to bucket zero, no matter what! Simplicity at its finest.

What is the runtime of any lookup operation in terms of N, the number of items in a map using this hash function? Use Θ notation.
Solution:
Θ(N)

Your friend's terrible hash function has turned what was supposed to be a hash map into a **glorified linked list**. Yikes!

c) After settling your disagreements, you and your friend decide to build a hash map together. Assume its a good one that works properly. This map resizes by a factor of 3 after exceeding a load factor of 1. Your friend is a bit antsy, and gives the following explanation for their worries:

"This seems weird. Usually this thing has constant runtime on an insert operation, but every now and then it has to resize which takes a *bunch* of operations. I'm not sure we can hit constant runtime *on average*."

You believe that this is possible. Using this page and the next, give a mathematical argument on why you're right. (**Hint**: Amortized analysis!). You can assume that insert always runs in constant time.

[Work space for problem 4c]

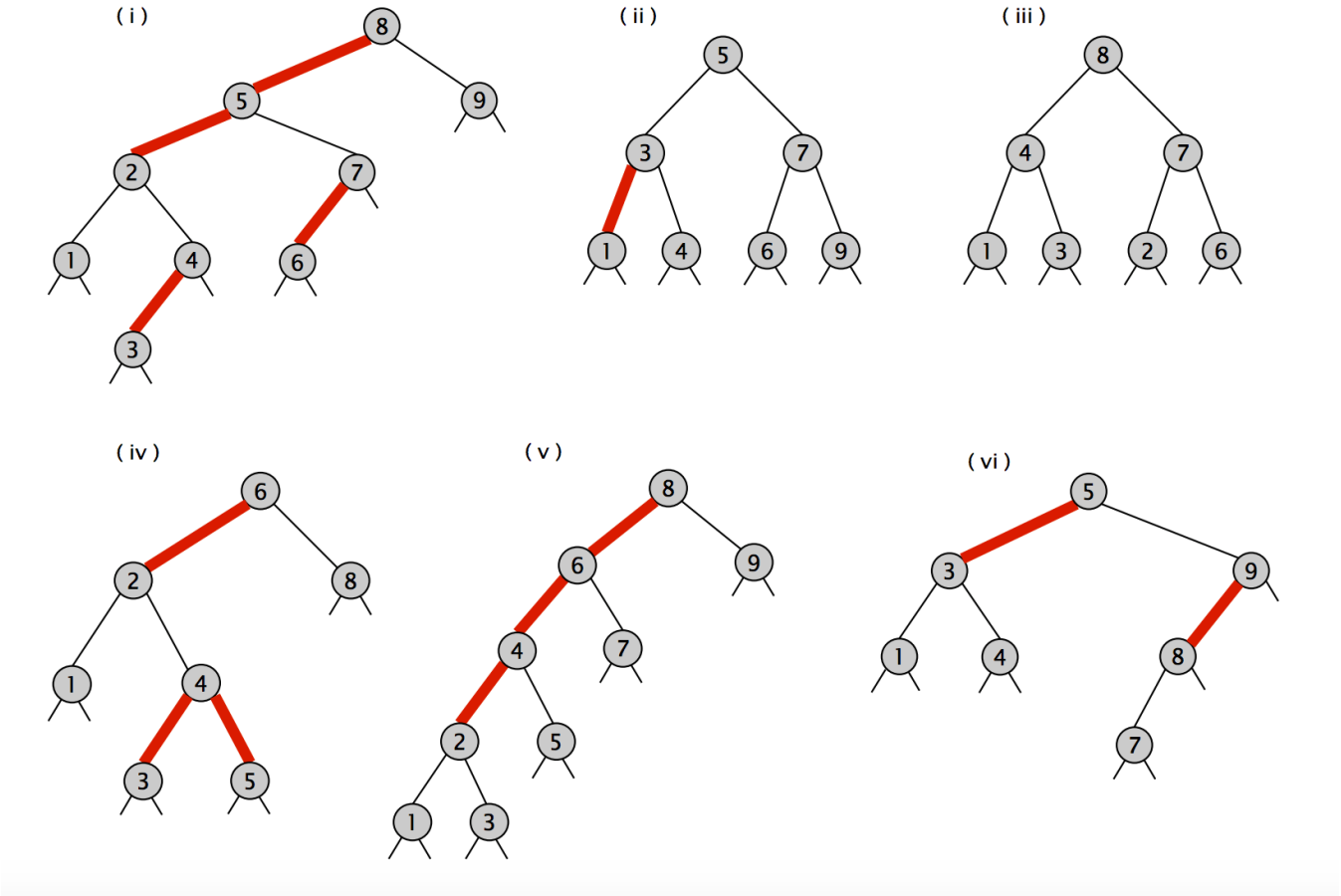
Solution:

Essentially in the lecture that Alan gave. Didn't have time to LaTEX it up yet, but I'll try to get to it soon. I did pretty much the same proof in discussion, if you still have your notes. Come to office hours if you want help on this before the solution goes up.

…IOU?

5. **I Like My Trees Bushy**

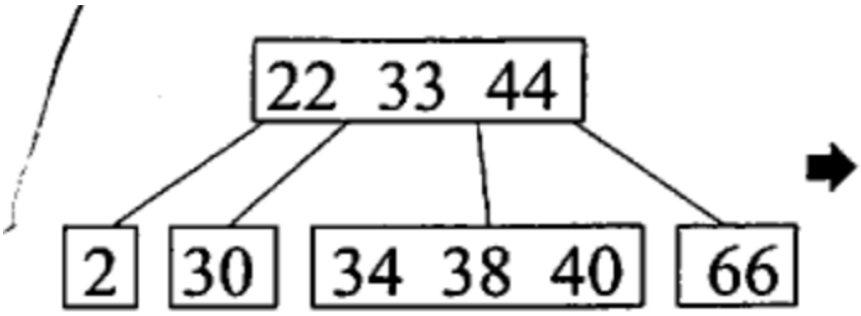
a) Which of the figures below represent legal left-leaning red-black trees?



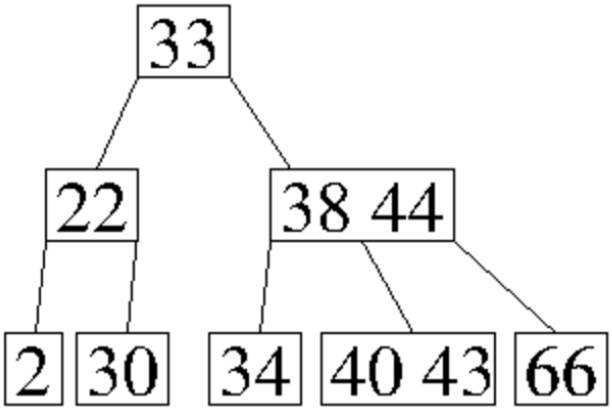
Solution:
None are legal. (i) because there can't be two red links in a row. (ii) because the path from the root to the leftmost leaf has fewer blank links than the path from the root to the rightmost leaf, (iii) because the keys are not in symmetric order, (iv) because there is a right-leaning red link, (v) because there are 3 red links in a row, and (vi) because the path from the root to the leftmost leaf has fewer black links than the path from the root to the leaves hanging off 7.

Note: Initially the solution said that (i) was legal. This was true in COS226 at Princeton, but not true in 61B. I did not realize that Josh had changed the LLRB isometry to 2-3 trees (instead of 2-3-4 trees)!

b) Draw the following B-Tree (2-3-4 Tree) after you perform insert(43):



Solution:



Note: Many students provided answers in which some of the leaves are at a greater depth than other leaves. If you learn only one thing about 2-3-4 trees, know that all the leaves must be at the same depth!

c) The smallest number of keys in a valid 2-3-4 Tree of height 3 (where the root is at height 0) is…..?

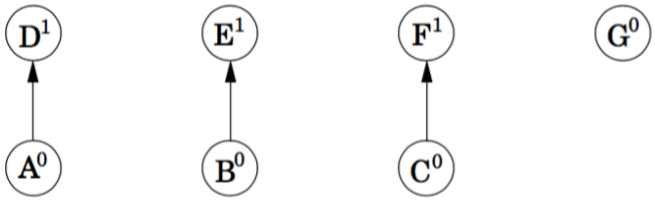
Solution:
15.

d) You start with the following Disjoint Set (also known an Union-Find) data structure:

Draw what the data structure looks like after the following sets of operations:

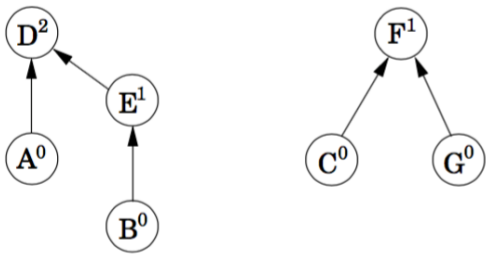
`union`(A,D),`union`(B,E),`union`(C,F)

Solution:



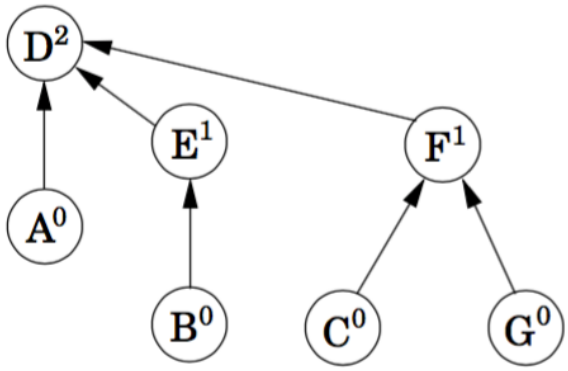
`union`(C, G),`union`(E, A)

Solution:



`union`(B, G)

Solution:



Note: You can ignore the superscripts on the letters- this is a problem stolen from the CS170 book and they have different notation. Solutions that broke ties different in the first two parts are also correct.