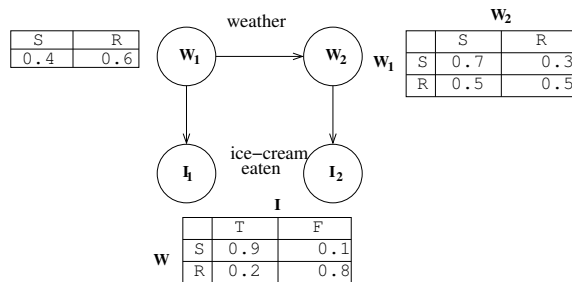


CS188 Fall 2015 Section 7: Sampling and Markov Models

Sampling

The Bayes Net in the diagram below describes a person's ice-cream eating habits. The nodes W_1 and W_2 stand for the weather on days 1 and 2, which can either be rainy R or sunny S . The nodes I_1 and I_2 stand for whether or not the person ate ice-cream that day. They can take the values T or F . The conditional probability distributions relevant to the graphical model are also given to you, note that there is a single conditional probability distribution $P(I|W)$, which I_1 and I_2 follow.



Suppose we use prior sampling to produce the following samples from the weather/ice-cream model:

(W_1, I_1, W_2, I_2)	(W_1, I_1, W_2, I_2)
R, F, R, F	R, F, R, F
S, F, S, T	S, T, S, T
S, T, S, T	R, F, R, T
S, T, S, T	S, T, R, F
S, T, R, F	R, F, S, T

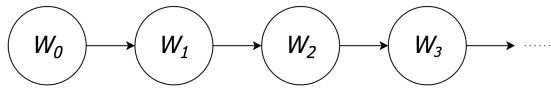
1. What is $\hat{P}(W_2 = R)$?
2. Cross off samples rejected by rejection sampling if we are computing $\hat{P}(W_2|I_1 = T, I_2 = F)$
 Now use likelihood weighting, and assume we've generated the following samples, given the evidence $I_1 = T$ and $I_2 = F$.

(W_1, I_1, W_2, I_2)	(W_1, I_1, W_2, I_2)
S, T, R, F	S, T, S, F
R, T, R, F	S, T, S, F
S, T, R, F	R, T, S, F

3. What is the weight of the first sample (S,T,R,F) above?
4. Use likelihood weighting to estimate $\hat{P}(W_2|I_1 = T, I_2 = F)$

Markov Model

We want to represent Berkeley's weather by Markov Model where we assume that the weather at day t is independent to the weather at day $0, \dots, t - 2$ given the weather at day $t - 1$. We have the following initial distribution and transition model.



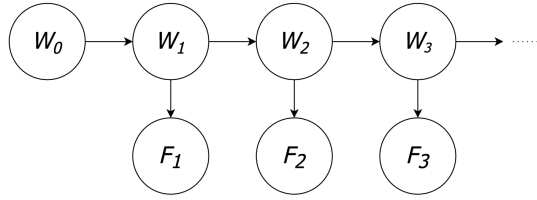
W_0	$P(W_0)$	W_i	W_{i-1}	$P(W_i W_{i-1})$
sun	0.9	sun	sun	0.8
rain	0.1	rain	sun	0.2
		sun	rain	0.3
		rain	rain	0.7

Compute the following.

1. $P(W_1 = sun)$
2. $P(W_2 = sun)$
3. $P(W_\infty = sun)$ (stationary distribution)

Hidden Markov Model

You are stuck on the second floor of Soda Hall working on the project for days. You have decided to not leave the building until you finish the project but wants to know what the weather is like outside. The only source of information you have is the thermometer in the room. We have a following initial distribution, transition model, and sensor model. Our initial distribution and transition model are same as the last problem.



W_0	$P(W_0)$	W_i	W_{i-1}	$P(W_i W_{i-1})$	F_i	W_i	$P(F_i W_i)$
sun	0.9	sun	sun	0.8	high	sun	0.7
rain	0.1	rain	sun	0.2	low	sun	0.3
		sun	rain	0.3	high	rain	0.2
		rain	rain	0.7	low	rain	0.8

1. We observed that $F_1 = high$ and computed $P(W_1)$ from previous section, what is the updated probability $P(W_1 = sun|F_1 = high)$?
2. We want to predict tomorrow's weather based on our observation. What is $P(W_2 = sun|F_1 = high)$?
3. Given $P(w_i|f_1, \dots, f_i)$ for all w_i , find $P(W_{i+1}|f_1, \dots, f_i)$.
4. **Forward Algorithm:** Given $P(w_i|f_1, \dots, f_i)$ for all w_i and new observation f_{i+1} , find $P(W_{i+1}|f_1, \dots, f_{i+1})$.