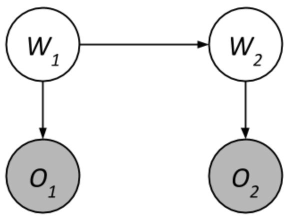
CS188 Spring 2016 Section 9: HMMs

Consider the following Hidden Markov Model.



X_1	$P(X_1)$
0	0.3
1	0.7

X_t	X_{t+1}	$P(X_{t+1}\|X_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

X_t	O_t	$P(O_t\|X_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5

Suppose that we observe $O_1 = A$ and $O_2 = B$.
Using the forward algorithm, compute the probability distribution $P(X_2|O_1 = A, O_2 = B)$ one step at a time.

1. Compute $P(X_1, O_1 = A)$.

   $P(X_1, O_1 = A) = P(X_1)P(O_1 = A|X_1)$
   $P(X_1 = 0, O_1 = A) = (0.3)(0.9) = 0.27$
   $P(X_1 = 1, O_1 = A) = (0.7)(0.5) = 0.35$

2. Using the previous calculation, compute $P(X_2, O_1 = A)$.

   $P(X_2, O_1 = A) = \sum_{x_1} P(x_1, O_1 = A)P(X_2|x_1)$
   $P(X_2 = 0, O_1 = A) = (0.27)(0.4) + (0.35)(0.8) = 0.388$
   $P(X_2 = 1, O_1 = A) = (0.27)(0.6) + (0.35)(0.2) = 0.232$

3. Using the previous calculation, compute $P(X_2, O_1 = A, O_2 = B)$.

   $P(X_2, O_1 = A, O_2 = B) = P(X_2, O_1 = A)P(O_2 = B|X_2)$
   $P(X_2 = 0, O_1 = A, O_2 = B) = (0.388)(0.1) = 0.0388$
   $P(X_2 = 1, O_1 = A, O_2 = B) = (0.232)(0.5) = 0.116$

Let's try to use Particle Filtering to estimate the distribution of $P(X_2|O_1 = A, O_2 = B)$.
We start with two particles: $P_1 = 0, P_2 = 1$. Use the following random numbers:

$$\{0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96\}$$

1. **Observe**: Compute the weight of the two particles after evidence $O_1 = A$.

   $w(P_1) = P(O_t = A|X_t = 0) = 0.9$
   $w(P_2) = P(O_t = A|X_t = 1) = 0.5$

2. **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

   We now sample from the weighted distribution we found above. After normalizing the weights, we find that P_1 maps to range [0, 0.643), and P_2 maps to range [0.643, 1). Using the first two random samples, we find:
   $P_1 = sample(weights, 0.22) = 0$
   $P_2 = sample(weights, 0.05) = 0$

3. **Elapse Time**: Now let's compute the elapse time particle update. Sample P_1 and P_2 from applying the time update.

   $P_1 = sample(P(X_{t+1}|X_t = 0), 0.33) = 0$
   $P_2 = sample(P(X_{t+1}|X_t = 0), 0.20) = 0$

4. **Observe**: Compute the weight of the two particles after evidence $O_2 = B$.

   $w(P_1) = P(O_t = B|X_t = 0) = 0.1$
   $w(P_2) = P(O_t = B|X_t = 0) = 0.1$

5. **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

   Because both of our particles have $X = 0$, resampling will still leave us with two particles with $X = 0$.
   $P_1 = 0$
   $P_2 = 0$

6. What is our estimated distribution for $P(X_2|O_1 = A, O_2 = B)$?

   $P(X_2 = 0|O_1 = A, O_2 = B) = 2/2 = 1$
   $P(X_2 = 1|O_1 = A, O_2 = B) = 0/2 = 0$