CS188 Spring 2016 Section 9: HMMs

Consider the following Hidden Markov Model.

X_1	$P(X_1)$
0	0.3
1	0.7

X_t	X_{t+1}	$P(X_{t+1}|X_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

X_t	O_t	$P(O_t|X_t)$
0	A	0.9
0	B	0.1
1	A	0.5
1	B	0.5
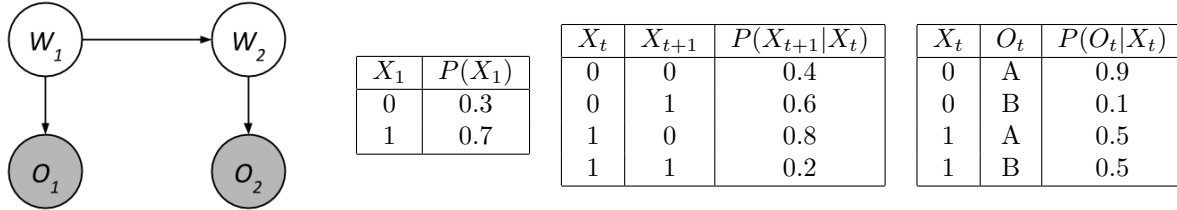
Suppose that we observe $O_1 = A$ and $O_2 = B$.
Using the forward algorithm, compute the probability distribution $P(X_2|O_1 = A, O_2 = B)$ one step at a time.

1. Compute $P(X_1, O_1 = A)$.

2. Using the previous calculation, compute $P(X_2, O_1 = A)$.

3. Using the previous calculation, compute $P(X_2, O_1 = A, O_2 = B)$.

Let's try to use Particle Filtering to estimate the distribution of $P(X_2|O_1 = A, O_2 = B)$.
We start with two particles: $P_1 = 0, P_2 = 1$. Use the following random numbers:

$$\{0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96\}$$

1. **Observe**: Compute the weight of the two particles after evidence $O_1 = A$.

2. **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

3. **Elapse Time**: Now let's compute the elapse time particle update. Sample P_1 and P_2 from applying the time update.

4. **Observe**: Compute the weight of the two particles after evidence $O_2 = B$.

5. **Resample**: Using the random numbers, resample P_1 and P_2 based on the weights.

6. What is our estimated distribution for $P(X_2|O_1 = A, O_2 = B)$?